Black Holes!

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Our slides are viewable here: https://goo.gl/Z3nkq7

1 What is a black hole?

In 2015, a huge scientific collaboration called LIGO discovered gravitational waves. Gravitational waves are huge ripples in spacetime that occur as a result of astrophysical events, like collisions between massive things in outer space. In 2015, the LIGO scientists found a signal that came from two black holes merging. This was huge scientific news, and resulted in the 2017 Nobel Prize in Physics for Rai Weiss, Kip Thorne, and Barry Barish.

Black holes are on the cutting edge of modern physics research, and there's a lot of new and fascinating physics about them. But it turns out, the basics of what a black hole is and what a black hole does are super understandable! And that's what this class is about.

So what are black holes? ***A black hole is a region of spacetime where the force of gravity is so strong that nothing, not even light, can escape.*** That's a lot of words, but the next two sections will go over some of the basics of relativity so that this definition makes sense.

2 Special relativity

The special theory of relativity was Einstein's first step towards revolutionizing the way we think about space, time, gravity, and more. It's a very famous theory with lots of applications and lots of strange paradoxical consequences. But if we go back to the very, very basics, it's not *all* that weird.

In our class, Emma threw a rubber duck at Anjali. Every observer – everyone paying attention – would agree that that event is something that happened. But not everyone will agree on *when* or *where* exactly it happened. In our class, Anjali also threw the duck at Emma, but this time Emma was running at her.

From Emma's point of view, that duck was coming at her really quickly. But for everyone else, it wasn't going that fast at all.

There's no real way to say one person's point of view is "more right" than someone else's. This is why we call it relativity – because the measurements that an observer can take for an event are *relative* to where they are and how fast they're going. This isn't a new idea. You've probably noticed that if you're driving on a highway the cars next to you don't seem to be going that fast, even if they're pushing 70mph. The cars going in the opposite direction, though, seem to be zooming by really quickly. It's because of how fast you're going!

So far, everything sounds reasonable. But things start to get bizarre when you get really fast – to be precise, almost as fast as the speed of light. The speed of light has been shown through a variety of experiments (look up the Michelson–Morley experiment if you're curious) to be a fundamental constant of the universe. The speed of light *does not change*, no matter how fast you're going. Whether you're standing on the ground, or chugging along on a train, or zipping around the universe in a fancy rocket ship at half the speed of light, you'd still measure the speed of light to be the same number: 299,792,458 m/s. This leads to some really weird stuff.

In class, we talked about a train. Say you're standing near some train tracks, killing time on a weekend. A train goes past you, and someone inside is killing time by bouncing a light beam off the ceiling of the train car. Each of you will see a different trajectory for the light, like in Figure 1:

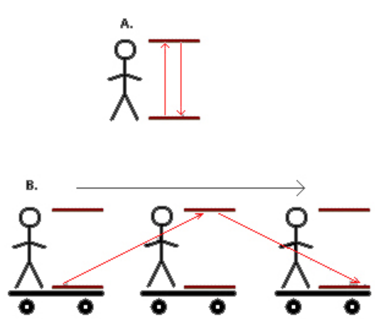


Figure 1: The person on the train sees a trajectory like A, where the light just bounces straight up and down over some height h. The person standing on the ground sees something more like B, with a triangular shape because of the motion of the train. The height of the triangle is still h, but it also covers some horizontal distance – so the person on the ground observes that the light has traveled a greater distance.

Let's think about what this means. We know that speed equals distance over time. But because the speed of light is constant (let's call it c, like a scientist would), we can write the following equation:

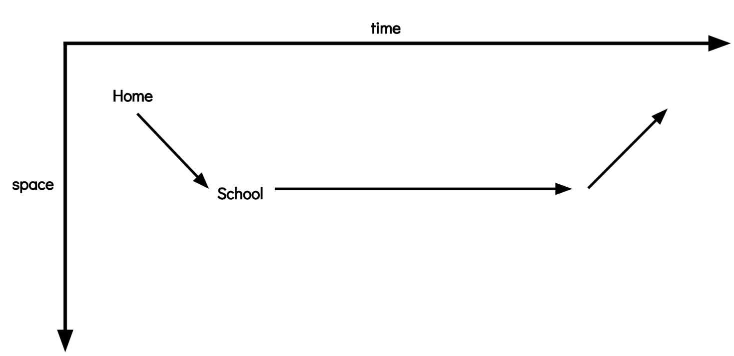
$$\frac{D}{t} = c = \frac{d}{t'}$$

where D is the distance the ground observer measures (we're calling it a capital D because it's bigger), and d is the distance the person in the train measures. The observer measures that the light took some amount of time to bounce – we're calling that t. The person in the car also measured a bounce time that we're calling t' (pronounced "t-prime", which is scientist for "a different t").

Why are these times different? The speed of light has to be the same. We know that that is true. We also know that the distances each observer measures are different. So, for each distance-over-time fraction to have the same value of c, the times each observer measures *have* to be different. Specifically, the observer in the train measures *less time* than the person on the ground. If each person had a stopwatch that they started when the beam turned on and stopped when the beam hit the floor again, the readings would be different! This is super weird.

What this means is that the speed you're moving at changes how fast time 'ticks'. The faster you're moving, the slower time goes. And it's not just time that's weird: in this example, we used the fact that the speed of light was the same and the distances were definitely different to show that the times had to be different. You can do this backwards and use a situation where the times are definitely different to show that distances are different! So it turns out that how you perceive space and time is highly dependent on how fast you're moving and where you are. And there's no way to say that one person's perception is better or more correct than another person's – everything is relative.

So space on its own and time on its own don't really have a clear meaning. It turns out, though, that if you do a little bit of fancy math, a combination of the two called *spacetime* does mean something no matter where you are or how fast you're moving. We use these drawings called "spacetime diagrams" to talk about this combination of space and time. This is the one we used in our class:



We have space on one axis – we're just worrying about one dimension for now. (Everyone in this world lives on a line, I guess.) The other axis is time. If you wake up in the morning and go to school from your house, you have to travel some distance in space, and also some amount of time. That's why the line from Home to School in the drawing is diagonal! Then you stay in school for the rest of the day, so there's a horizontal line where time is passing but you're not moving in space. And then you go home – another diagonal line, because you're moving through space and time again. These trajectories have a fancy name: physicists like to call them "worldlines".

One last thing: the same way that space and time are linked, energy and mass are linked. That's what Einstein's famous equation is all about: $E = mc^2$. E is energy, m is mass, and c is the speed of light. It actually turns out there's an extra factor in there, called γ (pronounced "gamma"), so the equation is actually $E = \gamma mc^2$. The γ factor has to do with corrections to the relationship between mass and energy depending on how fast you're going – a lot of relativity comes back to that. For example, γ gets really big if you're going near the speed of light (by really big, we mean close to infinity). What this equation then says is that if you're going at or faster than the speed of light, things seem to have infinite energy, which isn't possible. So that's part of why it isn't possible to go faster than the speed of light – the math just breaks.

And that's special relativity! It turns out a lot of the things we normally perceive as separate – time and space, mass and energy – are linked inextricably. This has huge consequences for the way a lot of the universe works. But Einstein wasn't done there! Next up, general relativity.

3 General relativity

General relativity is what happens when you look at how mass and energy interact with space and time. It turns out that the distribution of mass and energy causes spacetime to curve, and the curvature of spacetime causes the distribution of mass and energy to change.

Imagine you're standing on a trampoline with a bunch of bouncy balls – they'll all roll toward you. What if you and a friend stood next to each other? They'd roll toward you with even more acceleration. What if you and a friend stood on opposite sides of the trampoline? Things might start to get complicated.

Thought experiments like this are a good way to start picturing how spacetime interacts with mass and energy, if you and the balls are massive objects and the trampoline is like spacetime (but with just two spatial dimensions).

In three-dimensional spacetime, objects with mass and energy stretch and bend space like you can stretch and bend trampoline fabric. And when objects move, they will follow the shortest possible path along the curved spacetime – this path is called a geodesic. Imagine having a very massive object on your trampoline, and rolling a much smaller ball towards it. The path that the ball takes won't be straight anymore. It'll curve and dip because of the way that the heavy object has bent the trampoline sheet. This is more or less how the curvature of spactime affects the motion of masses. The orbit of one object around another, for example, is just a result of the way that the central object curves spacetime.

It turns out that, in this picture, gravity isn't really a force anymore. It's actually exactly equivalent to the acceleration along the shortest possible path. So when we talk about relativity, we don't talk about forces; we'll just talk about geometry – the way spacetime is squeezed and stretched because of where mass and energy are.

Einstein formalized all this with a famous equation called the Einstein field equation:

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta} \tag{1}$$

$G_{\alpha\beta}$ is a term that wraps up our description of the curvature and geometry of spacetime. $T_{\alpha\beta}$ is a term that wraps up our description of the distribution of mass and energy. The stuff in the middle is just a magical constant that ensures everything matches up properly.

We can see from this equation that spacetime curvature affects mass/energy distribution affects spacetime curvature – this interdependence is called back-propagation and it makes this equation really hard to solve. Einstein feared that no one would find any nontrivial solutions to this equation.

Thankfully, it turned out that there are some interesting exact solutions to the Einstein field equation. And most of them are black holes!

4 What are black holes like?

Now that we've talked about relativity, we've set the stage for black holes. We've talked about how mass and energy warp spacetime. A black hole is basically what happens when you have *a lot* of mass in a *really tiny* amount of space. Using our description of spacetime as a sheet, Figure 2 shows what a black hole does. At some point, you've fallen so far into the black hole there's no way you can get out, not even if you're light.

Now that (hopefully) you understand what a black hole is, let's talk about parts of a black hole. They basically consist of two main parts: an *event horizon* and a *singularity* (see Figure 3).

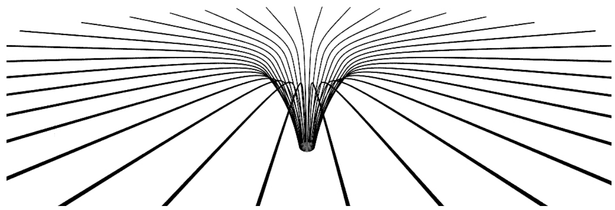


Figure 2: A black hole creates a tremendous amount of curvature because it's a very very heavy object that only physically takes up a very very tiny amount of space.

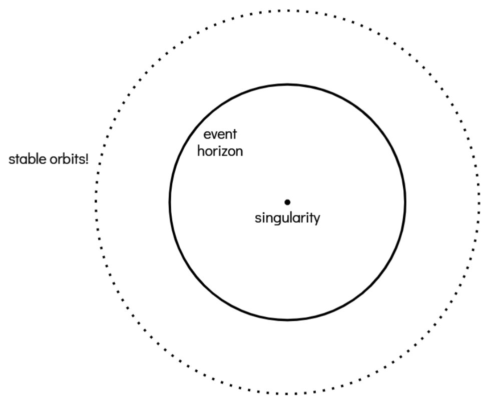


Figure 3: A simplified diagram of what's going on with a black hole.

The event horizon is the point of no return for a black hole. (More like the circle of no return. Maybe the sphere of no return? You get it.) It's a very well-defined radius that's mostly dependent on the mass inside the black hole.

Basically, as long as you're outside the event horizon of the black hole, it'd feel like being around any other massive object – like a star, or a planet, or a moon. But once you cross the event horizon, it is physically impossible for you to turn around. You could go as fast as light, but you wouldn't be able to reverse out of there. Trying to get out of a black hole once you've fallen past the event horizon is like asking time to run backwards. It just doesn't work. It's important to remember that the event horizon isn't a physical thing. There's no ring of fire or flashing light that'll let you know that you're about to cross this threshold.

At the center of the black hole is the *singularity*. This is (presumably) where all the mass goes once it's sucked in. It's an infinitesimally tiny, tiny, tiny, tiny,

tiny, tiny point at the center of the black hole. This is where all the math breaks and we have no concrete description of what's happening. Physicists have all sorts of theories, but we really have no idea.

A black hole is *not* like a vacuum. You won't get sucked in towards the singularity any more than a planet will suck you towards its center – unless you're past the event horizon. If you're watching someone get closer to a black hole, it'll just look like they're inching closer and closer to the event horizon. If you were in a rocket ship traveling towards a black hole, nothing would happen to indicate that you had passed the event horizon until you got sufficiently close that the forces of gravity would start to rip things apart.

Since not everything gets sucked into a black hole, it's possible for things to orbit black holes! These rings of orbiting stuff are called *accretion disks.* All sorts of stuff can build up around a black hole: dust, gas, rock, even light. And they can orbit the black hole stably – that is, they won't get sucked in or launched out. Light is kind of special (as you might have noticed by now). Since it can only go at one speed, it can only occupy one specific orbit. This is called the "innermost stable circular orbit", or the ISCO for short. The black holes at the center of galaxies (which we'll talk about later) also have all kinds of stuff orbiting them! Stars, planets, space dust, hot gas, and more.

So what can we know about a black hole? It turns out there are three main properties that we can measure: mass, charge, and spin. Black holes have a well-defined mass that can be measured by observing how it bends the paths of objects and light. (When light is bent around a massive object – this occurs because the object curves spacetime and thus also the trajectory of the light – you get all sorts of weird optical effects. This phenomenon is called *gravitational lensing.* Look it up! There are some really cool photos.) A black hole can have charge – we haven't observed any charged black holes in real life, but it's theoretically possible. Black holes can also spin, the same way the Earth spins about its axis. These three properties are the only things we can find out about a black hole, because to know anything else we'd have to go past the event horizon – but then we could never come out, or send any information out (if a tree falls in a forest...). This idea that only these three properties are knowable is called the *no-hair theorem.*

5 Formation and types of black holes

Where do black holes in real life come from? Well, there's a couple different options.

The most well-understood avenue of black hole formation creates stellar black holes. During their lives, the radius of a star is maintained via a careful balancing act of enormous outward pressure from the heat and radiation generated by nuclear fusion competing with the huge force of gravity from the star's mass pulling inward.

But a star can't sustain nuclear fusion forever. A star will fuse all its hydrogen into helium, all its helium into lithium, and further on through the periodic table until its core is made of iron. Fusing iron into cobalt (the next element) costs more energy than it creates, so fusion stops. Without the outward pressure from fusion, the star will begin to collapse inward under the force of gravity.

A star the size of our sun collapses until the pressure inside the nuclei of atoms becomes strong enough to win against gravity, leaving either a white dwarf or a neutron star. But stars with 5-10 times the mass of our sun collapse with so much force that nothing, not even the pressure inside of neutrons, can stop it. The star ejects a vast quantity of hot matter in a brilliant supernova, creating a nebula of stardust, and collapses into a stellar black hole.

Stellar black holes typically have spin, because the original star was spinning. Their event horizons have a small radius, and the gravity around the black hole is extreme due to the small size. If you fell into one, the gravity at your feet would be much stronger than the gravity at your head, and you would be stretched out. This is called "spaghettification."

In this universe, we have also observed supermassive black holes. These monsters are usually about 1000 times the mass of our sun, and we don't really know where they come from. Almost all large galaxies probably have a supermassive black hole at their centers, including the Milky Way.

Though they sound more dramatic, supermassive black holes are actually less extreme than stellar black holes in some respects. Because they're so massive, their event horizons have a very large radius, so they seem more diffuse. If the event horizon were an actual surface and you had enough water, a supermassive black hole would float in water.

6 Black hole weirdness

We've talked about what astrophyical black holes are really like and how they've been observed to behave. But because of all the weirdness that happens near and inside them, black holes are also fascinating theoretical laboratories for physicists to explore the extremes of what physics allows.

You've probably heard of wormholes – they show up in science fiction movies and stories all the time. But what are they, really? A wormhole is a theoretical structure that links two different points in space and time. It's like a tunnel with two ends, each of which opens up in a different part of the universe (or in different parts of different universes, maybe). The tunnel part is called an Einstein-Rosen bridge. Remember, though, wormholes haven't been proven to exist. We have no evidence that they actually happen, but nothing in the mathematics of space and time totally rules them out either.

There's an idea that each black hole can be treated as a wormhole into another universe (or maybe just a faraway unobservable part of our own universe), and that all the mass and energy absorbed by a black hole is spewed out on the other end of this wormhole, where it forms a "white hole". There are a lot of fun such theories which are fascinating to read about and ponder, but ultimately we don't conclusively have any evidence for any of them.

Hawking radiation is another cool black hole concept. It's named after Stephen Hawking, who came up with this idea in the 1970s. Hawking applied quantum field theory (a framework for quantum mechanics that was at the time fairly new) to black holes, and found that black holes can actually lose and emit mass. He found that black holes can have something like a temperature that's inversely proportional to their mass – the heavier the black hole, the 'colder' it is. So a small black hole (with approximately the mass of the Earth, for example), would have a high 'temperature'. Hot objects radiate energy in the form of heat, and similarly, black holes can radiate energy. But because energy and mass are kind of the same (remember $E = mc^2$? it comes in handy here), this is like the black hole losing mass. We say that the black hole dissipates, or evaporates.

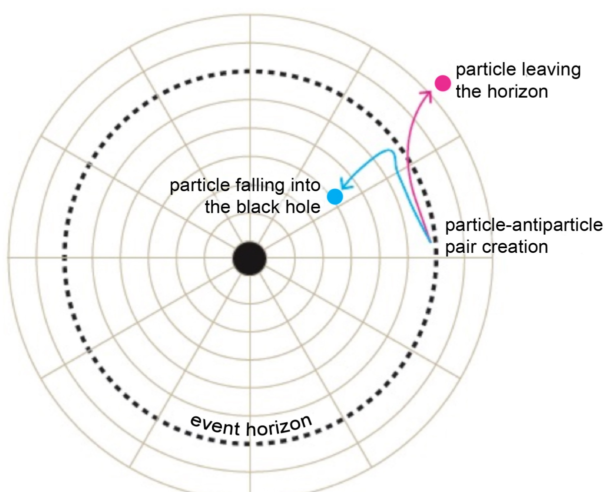


Figure 4: A simplified diagram of vacuum fluctuations and Hawking radiation. This is not entirely exact since it kind of shows a particle leaving once it's already inside the event horizon, but the paragraph after this clarifies what's going on.

Another way to think about Hawking radiation – this is a more exact description, but somewhat harder to visualize – is to talk about something called *vacuum fluctuations*. In addition to matter, the universe also contains antimatter. Antimatter particles have the same mass as their corresponding 'regular' matter particles, but they have the opposite value for various properties – what this means is that when matter and antimatter collide, they annihilate and release energy. What quantum field theory tells us is that sometimes pairs of matter and antimatter particles (called "virtual pairs") will pop into existence and then cancel each other out, release some energy, and disappear. But what happens if one of these pairs occurs and straddles the event horizon of a black hole? Remember, there's no physical barrier preventing that from happening. You could have the matter particle outside the event horizon, where it would be unaffected by the black hole's pull and could leave. But the antimatter particle could be inside the event horizon, where it would be necessarily sucked in towards singularity. At singularity, the antimatter would cancel out some of the matter already there, reducing the overall mass of the black hole.

7 LIGO

We started out this class talking about LIGO – the Laser Interferometer Gravitational-Wave Observatory. Gravitational waves occur when highly energetic events occur in spacetime. Two objects orbiting each other, for example, create ripples in spacetime. This is kind of like how if you dangled a stick in a pond and moved it around, the water would ripple around it. These ripples move through space and distort it, but they're normally too small for us to measure on Earth. But if something *really* energetic happens – the collision of two heavy objects like black holes, maybe – the amount of energy released is *huge*, and it manifests as ripples in spacetime, squeezing and stretching things as they ripple outwards.

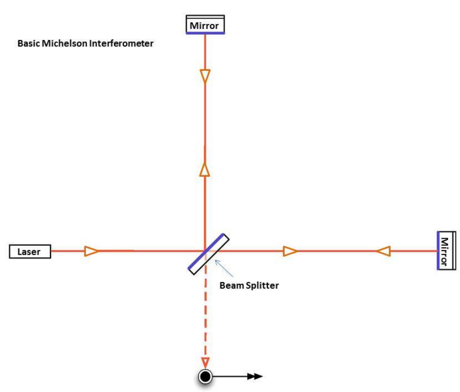


Figure 5: A simplified diagram of an interferometer.

LIGO uses a type of detector called an interferometer (see Figure 5) to look for these squeezes and stretches. An interferometer takes a laser beam and splits it along two perpendicular arms that end in mirrors. The light bounces back from the mirrors and recombines at a detector (the little black circle at the bottom in Figure 5). When spacetime is squeezed and stretched, the lengths of the two arms change and the light has to travel different distances. (The change in distance that we see on Earth is less than the diameter of an atom! LIGO is an engineering marvel because of how precisely and carefully we are able to measure this change in length.) This length change alters the way the two beams recombine at the detector, and you get interesting flickering/blinking patterns that tell you about how the arms were squeezed or stretched. And from this information, LIGO scientists are able to perform calculations that describe the huge astrophysical events that generated the tiny signals we are miraculously able to measure here on Earth.

8 In conclusion...

Black holes are fascinating objects for a number of reasons. They are direct consequences of Einstein's relativity, and don't require too much fancy math to understand. They are astrophysically significant and responsible for a lot of phenomena in outer space – the birth and death of stars, the stability of galaxies, and maybe even the development of the universe since the Big Bang. They are theoretical testbeds that allow physicists to explore the absolute extremes of what the laws of physics will allow. Plus, they're just really cool!

We hope you enjoyed this class and are inspired to learn more about black holes and all the amazing physics they're involved with. There was only so much we were able to go over in our class, but there's a whole universe of black hole research out there, and a lot of it can be quite accessible to you if you're interested. Please feel free to reach out to us if you have questions (our emails are at the top of this document), and thank you so much for letting us share this amazing science with you!